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A LAND USE PLAN DESIGN MODEL WITH FIXED INTERACTIONS AND CENTRAL FACILITY INTERACTIONS

by

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ABSTRACT

This article presents a combinatorial programming approach to land use plan design which is an extension of the Koopmans-Beckmann problem. Fixed interactions and central-facility interactions are taken into account. Computational experience is reported.

1. INTRODUCTION: FORMALIZING THE LAND USE PLANNING PROBLEM

A- The problem

The purpose of this article is to present a land use plan design model, and to report computational experience. It is a discrete model which attempts to find an optimal solution or at least a near-optimal solution when the objective function represents a tradeoff between site costs on one hand and a total transportation cost on the other hand. It is an extension of the Koopmans-Beckmann problem ([5]), previously proposed by this author ([7] , [8] , [9]) to solve the land use planning problem.

The Koopmans-Beckmann problem was chosen previously because it offered a very simple way to take into account the interactions between the location decisions of different activities in a combinatorial programming approach. However, as emphasized by L. Hopkins ([4]), the Koopmans-Beckmann approach to land use planning assumes that the trips between pairs of activities are fixed, which is a very strong and unrealistic assumption. In reality, the number of trips between pairs of activities varies with the distance, or more generally, the transportation cost between the activities, as is well known by urban transportation planners.

A class of models called location-allocation systems[1] seems to avoid that problem and therefore could be relevant to the land use plan design problem: given the location of certain demand activities (e.g.

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1 See refs (1), (11) and (12) for reviews and bibliographies on location theory algorithms in general and on location-allocation systems in particular.

residences), these models simultaneously locate central facilities (e.g. schools) and allocate demand activities to these central facilities. More precisely, a demand activity is allocated to the nearest central facility. This means that all trips between the demand activity considered and a central facility occur between the demand activity and the nearest central facility. There are no trips between the demand activity and facilities situated further away. However, unlike the Koopmans-Beckmann approach, location-allocation systems assume that a subset of activities, the demand activities, are already located. So they cannot be used directly to design a complete land use plan for a new town for instance. L. Hopkins (cf (4)) proposed an approach which uses iteratively an algorithm for a location-allocation model and a quadratic assignment algorithm to solve the land use plan design problem[1]. As the sequel of this paper will show, it is not at all necessary to use an algorithm for a location-allocation model.

The land use plan design model presented here can assign all activities, to all potential locations simultaneously, and considers both fixed interactions and central facility interactions. It is a very simple extension of the Koopmans-Beckmann problem and therefore allows the direct use of computational experience gained from that problem to solve the extended problem, without using any location-allocation algorithm.

1 His approach also uses decomposition to solve a very large size problem and takes into account air pollution. With the data that he uses he actually concludes that the quadratic assignment part is not worth using.

B- Methodology

No attempt was made to design an exact technique to solve the model proposed here. The explosive nature of combinatorial programming problems like the one presented here (or the Koopmans-Beckmann problem) makes exact techniques inefficient for practical problems (ref. 8). A heuristic technique of the hill-climbing type was implemented. This technique starts from an initial solution and improves on it systematically by using certain improvement rules. It stops when no further improvement is possible using the proposed rules. A local optimum has then been reached. In order to increase the probability of reaching a global optimum, several initial solutions can be generated at random. Another rationale for generating several local optima is that one can choose among the best local optima by use of a secondary criterion. Paragraph II presents the mathematical model, paragraph III presents the algorithm and reports computational experience. Paragraph IV concludes the article.

II. THE MATHEMATICAL MODEL

A- The model

We have N activities to assign to N locations. The set of activities is divided between two subsets: the subset {1,2,...,n} of activities which have fixed interactions among themselves and the subset of activities {n + 1,...,N} which have fixed interactions among themselves but which are such that the interactions between activities of the first subset and activities of the second subset can be zero or a given constant. Activities of

the first subset are called "demand" activities and activities of the second subset are called "facilities". Typically an activity of the first type could be a residence unit and an activity of the second type could be a school and all the trips to school would occur between the residence unit and the nearest school. Other facilities would include shopping centers, factories, recreation centers etc.

We assume that there are $m = N - n$ facilities divided among H types. Each demand activity is "allocated" to one and only one facility of a given type. This means that all the shopping trips are destined to one and only one shopping center and all the school trips to one and only one school. The problem to solve is to assign the N activities to the N locations and to allocate the n demand activities among the m facilities, so as to minimize a total cost composed of site costs and transportation costs. The assignment of activities to locations is represented by the matrix:

$$Y = [y_{ik}] \quad \text{with} \quad i = 1,\ldots,N \qquad k = 1,\ldots,N$$

where $$y_{ik} = \begin{cases} 1 & \text{if activity } i \text{ is assigned to location } k \\ 0 & \text{otherwise} \end{cases}$$

The allocation of demand activities to facilities is represented by the matrix.

$$X = [x_{if}] \quad \text{with} \quad i = 1,\ldots,n \qquad f = n+1,\ldots,N$$

where $$x_{if} = \begin{cases} 1 & \text{if the number of trips between } i \text{ and } f \text{ is non-zero} \\ 0 & \text{otherwise} \end{cases}$$

$$A = [a_{ik}] \qquad i = 1,\ldots,N \qquad k = 1,\ldots,N$$

is the matrix of site costs. a_{ik} is the site cost of assigning activity i to location k ; it can represent the sum of a development cost, a spatial congestion cost preventing building above a certain density limit, and an ecological cost taking into account the amenity or disamenity characteristics of assigning a particular activity to a particular location.

$$Q = [q_{ij}] \qquad \text{with} \qquad i = 1,\ldots,n \qquad j = 1,\ldots,n$$

is the matrix of interactions (i.e. trips) between the demand activities.

$$R = [r_{fg}] \qquad \text{with} \qquad f = n+1,\ldots,N \qquad g = n+1,\ldots,N$$

is the matrix of interactions between the facilities.

Let H be the set of indexes corresponding to the facilities of type $h(h = 1,\ldots,H)$.

$$K = [K_{ih}] \qquad \text{with} \qquad i = 1,\ldots,n \qquad h = 1,\ldots,H$$

is the matrix of interactions between demand activities and facilities. More precisely, K_{ih} is the number of trips between demand activity i and one facility of type h .

$$K' = [K_{hi}] \qquad \text{with} \qquad h = 1,\ldots,H \qquad i = 1,\ldots,n$$

is the matrix of trips between facilities and demand activities.

$$C = [c_{k\ell}] \qquad \text{with} \qquad k = 1,\ldots,N \qquad \ell = 1,\ldots,N$$

is the matrix of transportation costs between location pairs.

The cost function ϕ to be minimized is:

$$\phi(Y,X) = \sum_{i=1}^{N} \sum_{k=1}^{N} a_{ik}\, y_{ik} + \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{N} \sum_{\ell=1}^{N} q_{ij}\, c_{k\ell}\, y_{ik}\, y_{j\ell}$$

$$+ \sum_{f=n+1}^{N} \sum_{g=n+1}^{N} \sum_{k=1}^{N} \sum_{\ell=1}^{N} r_{fg}\, c_{k\ell}\, y_{fk}\, y_{g\ell}$$

$$+ \sum_{i=1}^{n} \sum_{h=1}^{t} \sum_{f \in H} \sum_{k=1}^{N} \sum_{\ell=1}^{N} (K_{ih} + K_{hi})\, x_{if}\, c_{k\ell}\, y_{ik}\, y_{f\ell}$$

We have the following constraints:

$$\sum_{i=1}^{N} y_{ik} = 1 \qquad k = 1,\ldots,N \qquad (1)$$

$$\sum_{k=1}^{N} y_{ik} = 1 \qquad i = 1,\ldots,N \qquad (2)$$

$$\sum_{f \in H} x_{if} = 1 \qquad i = 1,\ldots,n \qquad h = 1,\ldots,H \qquad (3)$$

Constraint (1) expresses that each location is the site of one and only one activity.

Constraint (2) expresses that each activity is assigned to one and only one location.

Constraint (3) expresses that each demand activity is allocated to one and only one facility of each type.

For a given matrix X , the problem reduces to a quadratic assignment problem of the Koopmans-Beckmann type and for a given Y the problem is decomposable into nH trivial subproblems; each subproblem requires the allocation of a demand activity to the closest facility of a given type. If only that part of Y were known which concerns the demand nodes, we would have a location-allocation problem to solve, i.e. we would have simultaneously to assign facilities to the remaining locations and to allocate the demand nodes to these facilities. So the model presented here can be considered either as an extension of location-allocation problems or as an extension of the Koopmans-Beckmann problem.

B- Assumptions Implicit in this Formulation

- The model is discrete and therefore the size of each activity must be determined before applying the model: a given land use, say open space, must be divided into a given number of discrete modules.
- It is a static model, which does not concern itself with the staging of the plan.
- A social rate of discount must be given to compute the development costs. This determines indirectly the tradeoff between the site costs and the total transportation cost.
- The transportation costs between location pairs are fixed which implies that the transportation system is given and that there is no congestion effect.

- This model focuses on accessibility, development costs and ecological costs but only insofar as the latter are independent of the relative location of the facilities. In particular we assume that there are no spillover effects such as air pollution.
- This model represents a very simple way but a very rigid one of introducing demand elasticity for travel in the Koopmans-Beckmann model. It is more relevant to school trips than to shopping trips. The latter are "partially substitutable" trips in L. Hopkins' terminology: this means that the interactions between a residential activity and shopping centers decline with distance (or in general transportation cost) but not in an all-or-nothing fashion (cf (2)).
- Finally, in this model there are no capacity constraints on the facilities, i.e. there is no limit to the total number of trips that can be destined to a given facility. One way to relax this assumption would be to introduce explicit capacity constraints in the model. For fixed Y, we would then have a transportation problem in integers: this would make the proposed algorithm much less efficient. Therefore another method was chosen: we use a penalty cost[1] to evaluate how violated the capacity requirements are by each local optimum produced by the algorithm. This gives a secondary criterion to choose among the local optima (cf appendix 1).

1 This penalty cost was also introduced by Lewis Hopkins in his own work at the suggestion of Britton Harris.

III. ALGORITHM

A- Description of the Algorithm

No efficient exact algorithm has yet been devised to solve the Koopmans-Beckmann problem (cf [5], [6], [8], [10]). A fortiori no efficient exact method exists to solve the model presented in the preceding paragraph, which is an extension of the Koopmans-Beckmann problem. One has therefore to resort to heuristic techniques. An efficient heuristic method (called ACTIV) was previously reported by this author for solving the Koopmans-Beckmann problem (cf [8] and [9]). It is a hill-climbing technique. The algorithm presented here (QUALF) is an extension of ACTIV. The idea is to solve iteratively for Y , X being fixed, and vice-versa.

More precisely QUALF works as follows:

1. Start with an initial assignment of activities to locations(obtained by random generation for instance).[1]
2. Allocate each demand activity to the nearest facility of each type.
3. Solve the resulting quadratic assignment problem by ACTIV.
4. Iterate on steps 2 or 3 until no further improvement is possible by step 2 or 3.

   Several local optima are generated by choosing at random the initial assignment of activities to locations.

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1 An alternative method of generating an initial plan would consist in randomly locating central facilities and then in solving the linear assignment problem obtained by neglecting fixed interactions, i.e. by considering only central facility interactions in the model. This was suggested to me by Lewis Hopkins.

8- Numerical Experiments

QUALF was implemented on the CDC Cyber 74 of the Université de Montréal and applied to test problems of different sizes which are described in Appendix 2. Site costs were all equal to zero and only interactions between demand activities and facilities were non-zero[1]. Table 1 and Figure 1 show the variation of the execution time with the total number of activities[2]. QUALF always improved on the initial cost by about 50%.

1 Most trip interactions in practice are of the demand activity - central facility type.

2 In the experiments reported in Figure 1, the initial activity allocations were always in the "natural" order {1,2,3,...,N}.

Problem Size: N	Number of Local Optima Generated	Range of Variation for the Execution Times in Seconds of CDC Cyber 74		
16	6	.08	to	.15
25	10	.30	to	.59
36	16	.92	to	2.14
49	20	2.36	to	5.12

Table 1

Execution Times for Problems of Different Sizes

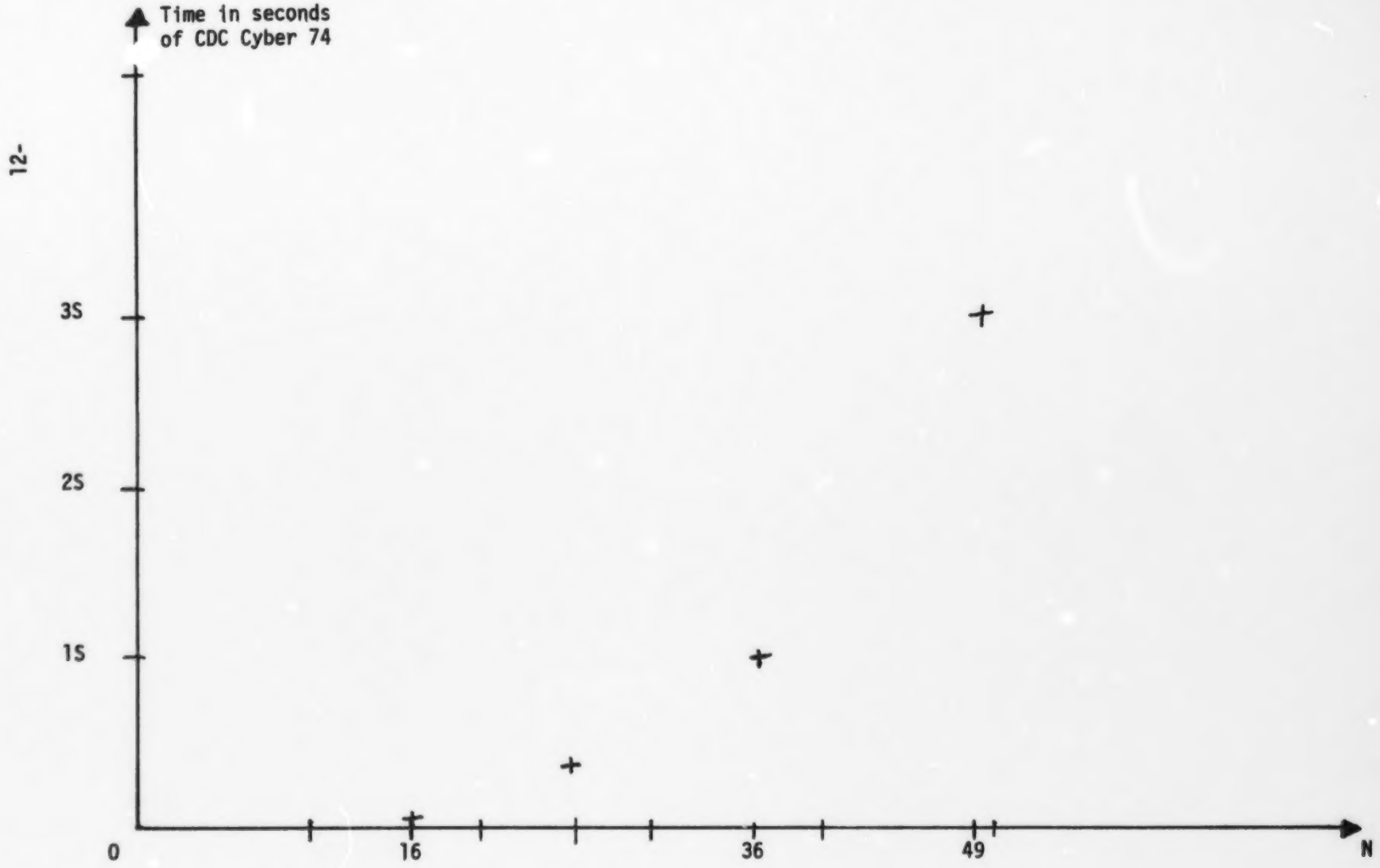

Figure 1

Variation of the Execution Time with the Total Number of Activities

Tables 2 and 3 describe in detail the best locally optimal solution of size N=49. The initial value of the objective function was 60503010 while the local optimum produced was 22075410.[1] This means an improvement of 63%. The penalty cost was 14738676.[2] The execution time was 3.579 s. The quadratic assignment subroutine was entered twice during the algorithm.

IV. CONCLUSION

The numerical experiments are encouraging: it was possible to solve a land use planning problem consisting in assigning 49 activities to 49 locations in less than 4 seconds on a CDC Cyber 74. However in order to solve very large size problems, it will be necessary to use a decomposition approach in combination with QUALF. In addition there would not be any difficulty at all in including air pollution interactions or incompatibility among land uses as does Lewis Hopkins, since we would only add new quadratic terms to the objective function but without changing the mathematical structure.

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1. However, since dummy interactions were introduced to force contiguity between certain modules the real value of the local optimum is only 8075410.

2. For the 20 local optima generated, this penalty cost varied between 4925380 and 53837260.

22	LDR 6	LDR 3	LDR 5	LDR 15	Industrial 35	Job Site 34
LDR 2	Neighborhood Center 39	LDR 1	LDR 11	Neighborhood Center 37	MDR 27	* 42
19	LDR 8	MDR 26	Community Commercial 29	MDR 24	LDR 12	* 41
47	45	46	High School 28	LDR 9	LDR 13	Office Job Site 30
49	48	Office Job Site 31	MDR 25	Neighborhood Center 38	LDR 14	23
20	LDR 10	* 44	* 43	LDR 4	18	Neighborhood Center 40
16	Neighborhood Center 36	LDR 7	17	21	Industrial 33	Job Site 32

REGIONAL RECREATION CENTER

* : Community Recreation Center
MDR: Medium Density Residential
LDR: Low Density Residential
: Estate Residential (no symbol)

Table 2

Land Use Plan for the Problem with 49 Activities

30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49
9	1	23	4		1	7	5	4	1	18	9	1	4	2	6	1	2	7	16
12	2		7		2	10	11	9	2	23	12	3	17	7	8	3	19	10	20
13	3		9		3	16	12	13	3		13	5	18	8		4	22	17	
14	4		10		5	17	15	14	6		14	6	21	10		5			
15	5		13		6	20	24	21	8		23	11	25	16		9			
18	6		14		8		27	25	19		24	15		19		11			
23	7		16		11				22			27		20		12			
24	8		17		12				26					22		13			
27	10		18		15									26		14			
	11		20		19											15			
	16		21		22											18			
	17		25		24											21			
	19				26											23			
	20				27											24			
	21															25			
	22															26			
	25															27			
	26																		

Table 3

Demand Activities Allocated to Facilities for the Problem of 49 Activities

(Columns represent the 20 facilities)

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Appendix 1

One of the hypotheses of the model embodied in QUALF is that central facilities have an unlimited capacity. This is only a first approximation. A penalty cost is derived here to evaluate the local optima generated by QUALF.

Let M_h be the expected capacity of a facility of type h, in trips, and ξ_h a weight expressing the relative importance of facility type h, Q_{ih} the total number of trips per unit of time between demand activity i and facility type h (in both directions).

The penalty cost P is defined by:

$$P = \sum_{h=1}^{t} \sum_{f \epsilon H} \xi_h \left[\sum_{i=1}^{n} Q_{ih} x_{if} - M_h \right]^2$$

In the numerical experiments, all facility types were given equal weights ($\xi_h = 1$).

Appendix 2

In the test problems, land uses were divided into modules of equal size to be assigned to locations of the same size situated in a square of 4, 5, 6 and 7 modules on each size respectively. For instance, for the problem of size 16, the locations were numbered as indicated in the following figure

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Each module is a square having a side of 100 units of length. The cost of travel is taken equal to the length. The trips were taken from "realistic" but hypothetical data used by L. Hopkins (ref. [4] p. 851) and are given in Table 4. The trip matrix is assumed symmetrical. Only trips between residential activities and other activities are non-zero, i.e. no trips between residential activities or between central facilities are considered. Site costs are assumed zero.

Dummy interactions of 5000 trips are assumed between two modules belonging to the same "real" activity so as to ensure contiguity.

Activity Type	Neighborhood Center: elementary school, recreation, retail	High School	Community Recreation	Regional Recreation	Community Commercial	Office Job Site	Industrial Job Site
Estate Residential	102	41	5	6	13	5	3
Low Density Residential	347	65	26	29	67	35	20
Medium Density Residential	414	49	70	78	120	71	39

Table 4

Demand in Trips Per Day from Each Activity Type to Each Activity Type

The activity modules for the different problem sizes are described below.

Problem with N = 16

Modules	1	to	5	:	low density residential
Modules	6	to	7	:	estate residential
Modules	8	to	9	:	medium density residential
Module	10			:	office job site
Modules	11	to	12	:	neighborhood centers
Modules	13	to	14	:	community recreation center (1 dummy module)
Modules	15	to	16	:	" " "

Problem with N = 25

Modules	1	to	8	:	low density residential
Modules	9	to	12	:	estate residential
Modules	13	to	14	:	medium density residential
Module	15			:	office job site
Modules	16	to	17	:	industrial job site (1 dummy module)
Modules	18	to	19	:	" " "
Modules	20	to	21	:	neighborhood centers
Modules	22	to	23	:	community recreation center (1 dummy)
Modules	24	to	25	:	" " "

Problem with N = 36

Modules	1	to	12	:	low density residential
Modules	13	to	18	:	estate residential
Modules	19	to	22	:	medium density residential
Module	23			:	office job site
Modules	24	to	25	:	industrial job site (1 dummy)
Modules	26	to	27	:	industrial job site (1 dummy)
Modules	28	to	32	:	neighborhood centers
Modules	33	to	34	:	community recreation center (1 dummy)
Modules	35	to	36	:	" "

Problem with N = 49

Modules	1	to	15	:	low density residential
Modules	16	to	23	:	estate residential
Modules	24	to	27	:	medium density residential
Module	28			:	high school
Module	29			:	community commercial
Modules	30	to	31	:	office job sites
Modules	32	to	33	:	industrial job site (1 dummy)
Modules	34	to	35	:	" "
Modules	36	to	40	:	neighborhood centers
Modules	41	to	42	:	community recreation center (1 dummy)
Modules	43	to	44	:	" "
Modules	45	to	49	:	regional recreation (4 dummies)